A Yet Faster Version of Complex-valued Multilayer Perceptron Learning Using Singular Regions and Search Pruning

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Keywords:
Complex-valued multilayer perceptron, Learning method, Singular region, Search pruning.

Abstract:
In the search space of a complex-valued multilayer perceptron having J hidden units, C-MLP(J), there are singular regions, where the gradient is zero. Although singular regions cause serious stagnation of learning, there exist narrow descending paths from the regions. Based on this observation, a completely new learning method called C-SSF (complex singularity stairs following) 1.0 was proposed, which utilizes singular regions to generate starting points of C-MLP(J) search. Although C-SSF1.0 finds excellent solutions of successive C-MLPs, it takes long CPU time because the number of searches increases as J gets larger. To deal with this problem, C-SSF1.1 was proposed, a few times faster by the introduction of search pruning, but it still remained unsatisfactory. In this paper we propose a yet faster C-SSF1.3, going further with search pruning, and then evaluate the method in terms of solution quality and processing time.

1 INTRODUCTION

Complex-valued neural networks (Hirose, 2012) have the attractive features real-valued ones don't have. A complex-valued multilayer perceptron (C-MLP) can naturally represent a periodic and/or unbounded function, which is not easy at all for a real-valued MLP.

Among learning methods of C-MLPs, complex back propagation (C-BP) (Kim and Guest, 1990; Leung and Haykin, 1991) is basic and well-known. A higher-order learning method was proposed to get better performance (Amin et al., 2011). Complex Broyden-Fletcher-Goldfarb-Shanno (C-BFGS) (Suzumura and Nakano, 2013) finds nice solutions after many independent runs.

There exist flat subspaces called singular regions in the C-MLP search space (Nitta, 2013), as is the case with a real-valued MLP (Fukumizu and Amari, 2000). Singular regions have been avoided (Amari, 1998) because they cause serious stagnation of learning. However, they can be utilized as excellent initial points when we perform search for successive numbers of hidden units. This viewpoint led to the invention of a completely new learning method. Actually, a method called SSF (Singularity Stairs Following) (Satoh and Nakano, 2013) was proposed for real-valued MLPs, utilizing reducibility mapping (Fukumizu and Amari, 2000) and eigenvector descent (Satoh and Nakano, 2012). It stably and successively found excellent solutions.

Recently a complex version of SSF, called C-SSF 1.0, was proposed (Satoh and Nakano, 2014), utilizing complex reducibility mapping (Nitta, 2004), eigenvector descent, and C-BFGS. It stably found excellent solutions in C-MLP search space, whose solution quality was better than C-BFGS. However, it took several times longer than C-BFGS. To make C-SSF1.0 faster, C-SSF1.1 (Satoh and Nakano, 2015) was proposed by introducing search pruning. It ran a few times faster than C-SSF1.0 without losing the superb solution quality, but still remained unsatisfactory in processing time.

This paper proposes a yet faster version of C-SSF called C-SSF1.3 by introducing two contrivances: putting a ceiling on the number of

searches and utilizing multiple best solutions to generate starting points. Our experiments compare solution quality and processing time of the proposed C-SSF1.3 with those of C-SSF1.1, C-BP, and C-BFGS.

2 SINGULAR REGIONS

This section explains how singular regions are generated. Consider a complex-valued MLP with J hidden units, C-MLP(J), whose output is f_J.

$$f_J(\boldsymbol{x};\boldsymbol{\theta}_J) = w_0 + \sum_{j=1}^{J} w_j z_j, \quad z_j \equiv g(\boldsymbol{w}_j^T \boldsymbol{x}) \quad (1)$$

Here $\boldsymbol{\theta}_J = \{w_0, w_j, \boldsymbol{w}_j, j = 1, \cdots, J\}$ is a parameter vector. Input $\boldsymbol{x}$, weights $\boldsymbol{w}_j$, w_j, output f_J, and teacher signal y are all complex. Given data $\{(\boldsymbol{x}^\mu, y^\mu), \mu = 1, \cdots, N\}$, we want to find $\boldsymbol{\theta}_J$ minimizing the following.

$$E_J = \sum_{\mu=1}^{N} \delta^\mu \overline{\delta^\mu}, \qquad \delta^\mu \equiv f_J(\boldsymbol{x}^\mu;\boldsymbol{\theta}_J) - y^\mu \quad (2)$$

Next, consider C-MLP($J-1$) with $J-1$ hidden units. Its output is f_{J-1}.

$$f_{J-1}(\boldsymbol{x};\boldsymbol{\theta}_{J-1}) = u_0 + \sum_{j=1}^{J-1} u_j v_j, \; v_j \equiv g(\boldsymbol{u}_j^T \boldsymbol{x}) \quad (3)$$

Here $\boldsymbol{\theta}_{J-1} = \{u_0, u_j, \boldsymbol{u}_j, j = 1, \cdots, J-1\}$ is a parameter vector of C-MLP($J-1$), and let the optimal $\boldsymbol{\theta}_{J-1}$ be $\widehat{\boldsymbol{\theta}}_{J-1}$.

Sussmann (Sussmann, 1992) pointed out the uniqueness and reducibility of real-valued MLPs. Much the same uniqueness and reducibility hold for complex-valued MLPs (Nitta, 2004). Now consider three reducibility mappings α, β, and γ; then, apply α, β, and γ to the optimal $\widehat{\boldsymbol{\theta}}_{J-1}$ to get $\widehat{\boldsymbol{\Theta}}_J^\alpha$, $\widehat{\boldsymbol{\Theta}}_J^\beta$, and $\widehat{\boldsymbol{\Theta}}_J^\gamma$ respectively.

$$\widehat{\boldsymbol{\theta}}_{J-1} \xrightarrow{\alpha} \widehat{\boldsymbol{\Theta}}_J^\alpha, \quad \widehat{\boldsymbol{\theta}}_{J-1} \xrightarrow{\beta} \widehat{\boldsymbol{\Theta}}_J^\beta, \quad \widehat{\boldsymbol{\theta}}_{J-1} \xrightarrow{\gamma} \widehat{\boldsymbol{\Theta}}_J^\gamma$$

$$\begin{aligned}
\widehat{\boldsymbol{\Theta}}_J^\alpha \;\equiv\; & \{\boldsymbol{\theta}_J | w_0 = \widehat{u}_0,\; w_1 = 0, \\
& w_j = \widehat{u}_{j-1}, \boldsymbol{w}_j = \widehat{\boldsymbol{u}}_{j-1}, j = 2, \cdots, J\} \quad (4) \\
\widehat{\boldsymbol{\Theta}}_J^\beta \;\equiv\; & \{\boldsymbol{\theta}_J | w_0 + w_1 g(w_{10}) = \widehat{u}_0, \\
& \boldsymbol{w}_1 = [w_{10}, 0, \cdots, 0]^T, w_j = \widehat{u}_{j-1}, \\
& \boldsymbol{w}_j = \widehat{\boldsymbol{u}}_{j-1}, j = 2, \cdots, J\} \quad (5) \\
\widehat{\boldsymbol{\Theta}}_J^\gamma \;\equiv\; & \{\boldsymbol{\theta}_J | w_0 = \widehat{u}_0, w_1 + w_m = \widehat{u}_{m-1}, \\
& \boldsymbol{w}_1 = \boldsymbol{w}_m = \widehat{\boldsymbol{u}}_{m-1}, w_j = \widehat{u}_{j-1}, \\
& \boldsymbol{w}_j = \widehat{\boldsymbol{u}}_{j-1}, j \in \{2, \cdots, J\} \setminus \{m\}\} \quad (6)
\end{aligned}$$

Now, we have the following singular regions.
(1) The intersection of $\widehat{\boldsymbol{\Theta}}_J^\alpha$ and $\widehat{\boldsymbol{\Theta}}_J^\beta$ forms singular region $\widehat{\boldsymbol{\Theta}}_J^{\alpha\beta}$, where only w_{10} is free. In the singular region the following hold:

$$w_0 = \widehat{u}_0,\; w_1 = 0,\; \boldsymbol{w}_1 = [w_{10}, 0, \cdots, 0]^T,$$
$$w_j = \widehat{u}_{j-1},\; \boldsymbol{w}_j = \widehat{\boldsymbol{u}}_{j-1},\; j = 2, \cdots, J.$$

(2) $\widehat{\boldsymbol{\Theta}}_J^\gamma$ is a singular region, where the following holds: $w_1 + w_m = \widehat{u}_{m-1}$, $m = 2, \cdots, J$.

After finishing learning of C-MLP($J-1$), C-SSF starts learning of C-MLP(J) from points in the singular region of C-MLP(J). Since the gradient is zero all over the singular region, the gradient won't give us any information in which direction to go. Thus we employ eigenvector descent (Satoh and Nakano, 2012). Picking up a negative eigenvalue, we have two search directions based on its eigenvector.

3 C-SSF

This section describes the former versions and the proposed version of C-SSF (Complex Singularity Stairs Following). C-SSF learns C-MLPs.

3.1 Basic Framework

The origin of C-SSF is C-SSF1.0 (Satoh and Nakano, 2014). C-SSF starts search from C-MLP($J=1$) and then gradually increases the number of hidden units J one by one until J_{max}. When searching C-MLP(J), the method applies reducibility mapping to the optimum of C-MLP($J-1$) to get two kinds of singular regions $\widehat{\boldsymbol{\Theta}}_J^{\alpha\beta}$ and $\widehat{\boldsymbol{\Theta}}_J^\gamma$. When starting search from the singular region, the method employs eigenvector descent (Satoh and Nakano, 2012), which finds descending directions, and from then on employs complex BFGS (C-BFGS). The general flow of C-SSF1.0 is given below. Let $\{w_0^{(J)}, w_j^{(J)}, \boldsymbol{w}_j^{(J)}, j = 1, \cdots, J\}$ denote parameters of C-MLP(J).

Here we give notes on the implementation used in our experiments. In Algorithm 1, p in steps 1.1 and 2.1.1 is free and was set to -1, 0, and 1. Moreover, q in step 2.2.1 is also free and was set to 0.5, 1.0, and 1.5, which correspond to internal division, boundary, and external division respectively. In Algorithm 2, the golden section search (Luenberger, 1984) was employed as a line search to find the suitable step length.

Algorithm 1 : C-SSF Method (ver 1.0 or 1.1)

step 1. Search for MLP(1)

1.1 Set an initial point on $\widehat{\boldsymbol{\Theta}}_1^{\alpha\beta}$:
$w_0^{(1)} \leftarrow \overline{y},\ w_1^{(1)} \leftarrow 0,\ \boldsymbol{w}_1^{(1)} \leftarrow [p, 0, \cdots, 0]^T$

1.2 **Search_from_singular_region**

1.3 Store the best as $\widehat{w}_0^{(1)}$, $\widehat{w}_1^{(1)}$, $\widehat{\boldsymbol{w}}_1^{(1)}$; $J \leftarrow 2$.

step 2. Search for MLP(J)

while $J \leq J_{max}$ **do**

2.1 Search from $\widehat{\boldsymbol{\Theta}}_J^{\alpha\beta}$:

2.1.1 **Set_an_initial_point_on_**$\widehat{\boldsymbol{\Theta}}_J^{\alpha\beta}$

2.1.2 **Search_from_singular_region**

2.2 Search from $\widehat{\boldsymbol{\Theta}}_J^{\gamma}$:

for $m = 2, \cdots, J$ **do**

2.2.1 **Set_an_initial_point_on_**$\widehat{\boldsymbol{\Theta}}_J^{\gamma}$

2.2.2 **Search_from_singular_region**

end for

2.3 Get the best among all solutions obtained in steps **2.1** and **2.2**, and store it as $\widehat{w}_0^{(J)}$, $\widehat{w}_j^{(J)}$, $\widehat{\boldsymbol{w}}_j^{(J)}, j = 1, \cdots, J$. Then, $J \leftarrow J + 1$.

end while

Algorithm 2 : Search_from_singular_region

step 1. Calculate the Hessian and get all the negative eigenvalues and their eigenvectors.

step 2.

for each negative eigenvalue with its eigenvector $\boldsymbol{u}$ **do**

2.1 Perform a line search in the direction of $\boldsymbol{u}$, start search using C-BFGS afterward, and keep the solution.

2.2 Perform a line search in the direction of $-\boldsymbol{u}$, start search using C-BFGS afterward, and keep the solution.

end for

Algorithm 3 : Set_an_initial_point_on_$\widehat{\boldsymbol{\Theta}}_J^{\alpha\beta}$

$w_0^{(J)} \leftarrow \widehat{w}_0^{(J-1)}$,
$w_1^{(J)} \leftarrow 0,\ \boldsymbol{w}_1^{(J)} \leftarrow [p, 0, \cdots, 0]^T$,
$w_j^{(J)} \leftarrow \widehat{w}_{j-1}^{(J-1)},\ \boldsymbol{w}_j^{(J)} \leftarrow \widehat{\boldsymbol{w}}_{j-1}^{(J-1)},\ j = 2, \cdots, J$

C-SSF has the following characteristics (Satoh and Nakano, 2014; Satoh and Nakano, 2015).
(1) The excellent solution of C-MLP(J) will be obtained one after another for J=1,$\cdots$,J_{max}. C-SSF guarantees that training error of C-MLP(J) is smaller than that of C-MLP($J-1$) since C-SSF descends in C-MLP(J) search space from the singular regions corresponding to the optimum of C-MLP($J-1$). This monotonic feature will be quite useful for model selection. However, such monotonic decrease of training error is not guaranteed for existing methods.
(2) C-SSF runs without using random number, meaning it always finds the same set of solutions.

Algorithm 4 : Set_an_initial_point_on_$\widehat{\boldsymbol{\Theta}}_J^{\gamma}$

$w_0^{(J)} \leftarrow \widehat{w}_0^{(J-1)}$,
$w_1^{(J)} \leftarrow q \times \widehat{w}_{m-1}^{(J-1)},\ \boldsymbol{w}_1^{(J)} \leftarrow \widehat{\boldsymbol{w}}_{m-1}^{(J-1)}$,
$w_m^{(J)} \leftarrow (1-q) \times \widehat{w}_{m-1}^{(J-1)},\ \boldsymbol{w}_m^{(J)} \leftarrow \widehat{\boldsymbol{w}}_{m-1}^{(J-1)}$,
$w_j^{(J)} \leftarrow \widehat{w}_{j-1}^{(J-1)},\ \boldsymbol{w}_j^{(J)} \leftarrow \widehat{\boldsymbol{w}}_{j-1}^{(J-1)}$,
$j \in \{2, \cdots, J\} \setminus \{m\}$

3.2 Search Pruning

C-SSF1.0 stably found excellent solutions, better than C-BFGS. However, it took several times longer than C-BFGS because the number of searches got larger and larger as the number of hidden units J increased. Thus, a faster version C-SSF1.1 (Satoh and Nakano, 2015) was proposed by introducing search pruning.

The general flow of C-SSF1.1 is the same as Algorithm 1 since search pruning is embedded in search using C-BFGS at steps 2.1 and 2.2 of Algorithm 2. Although search pruning is explained in detail in (Satoh and Nakano, 2015), the main point is shown below.

Let $\boldsymbol{\theta}^{(t)}$ and $\boldsymbol{\phi}^{(\tau)}$ be a current search point and a point stored during a previous search respectively. Since $\boldsymbol{d} = (\cdots, d_m, \cdots)^T$ is a normalizing vector, $\boldsymbol{v}^{(t)}$ and $\boldsymbol{r}^{(\tau)}$ are normalized points. The normalization is introduced to prevent any weight having a large absolute value from influencing the decision too much.

$$d_m \leftarrow \begin{cases} \left|\frac{1}{\theta_m^{(t-1)}}\right| & (1 < |\theta_m^{(t-1)}|) \\ & (|\theta_m^{(t-1)}| \leq 1) \end{cases} \quad (7)$$

$$\boldsymbol{v}^{(t)} \leftarrow \mathrm{diag}(\boldsymbol{d})\ \boldsymbol{\theta}^{(t)} \quad (8)$$

$$\boldsymbol{v}^{(t-1)} \leftarrow \mathrm{diag}(\boldsymbol{d})\ \boldsymbol{\theta}^{(t-1)} \quad (9)$$

$$\boldsymbol{r}^{(\tau)} \leftarrow \mathrm{diag}(\boldsymbol{d})\ \boldsymbol{\phi}^{(\tau)},\ \tau = 1, \cdots, T \quad (10)$$

Here $m = 1, \cdots, 2M$, where M is the number of complex weights. Let T be the number of points stored so far, and diag($\boldsymbol{d}$) is a diagonal matrix whose diagonal elements are $\boldsymbol{d}$.

See Figure 1. Now consider a line L_1 through two points $\boldsymbol{r}^{(\tau-1)}$ and $\boldsymbol{r}^{(\tau)}$, and a line L_2 through two points $\boldsymbol{v}^{(t-1)}$ and $\boldsymbol{v}^{(t)}$. Then consider a line

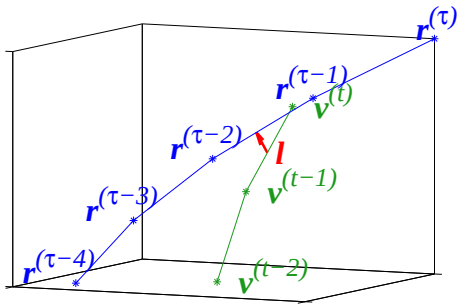


Figure 1: Conceptual Diagram of Search Pruning of C-SSF1.1

L_3 perpendicular to each of L_1 and L_2. Note that line L_3 includes the shortest line segment $\boldsymbol{\ell}$ between L_1 and L_2. Based on this $\boldsymbol{\ell}$, we decide whether the current search route is to merge onto a previous search route. We can calculate $\boldsymbol{\ell}$, which can be represented as below.

$$\boldsymbol{\ell} = (\boldsymbol{r}^{(\tau-1)} + a_1 \boldsymbol{\Delta r}^{(\tau)}) - (\boldsymbol{v}^{(t-1)} + a_2 \boldsymbol{\Delta v}^{(t)}) \quad (11)$$
$$\boldsymbol{\Delta r}^{(\tau)} \equiv \boldsymbol{r}^{(\tau)} - \boldsymbol{r}^{(\tau-1)}, \ \boldsymbol{\Delta v}^{(t)} \equiv \boldsymbol{v}^{(t)} - \boldsymbol{v}^{(t-1)}$$

By solving $\min_{\boldsymbol{a}} \ \boldsymbol{\ell}^{\mathrm{T}} \boldsymbol{\ell}$, unknown a_1 and a_2 can be determined. The following are the condition for $\boldsymbol{\ell}$ to start from a point between $\boldsymbol{v}^{(t-1)}$ and $\boldsymbol{v}^{(t)}$ and to end at a point between $\boldsymbol{r}^{(\tau-1)}$ and $\boldsymbol{r}^{(\tau)}$.

$$0 \le a_1 \le 1, \qquad 0 \le a_2 \le 1 \qquad (12)$$

If $\boldsymbol{\ell}$ does not satisfy the condition eq.(12) for any $\tau = 1, \cdots, T$, we consider the current search route does not merge onto any previous route. If the condition holds for a certain τ, however, we check whether the current search is to be pruned. The current search is pruned if the absolute value of each element of $\boldsymbol{\ell}$ is smaller than predefined ε.

Here implementation details in our experiments are described. Checking of search pruning and storing of current points are carried out at intervals of 100 search steps. Moreover, we set $\varepsilon = 0.3$ for a threshold of search route proximity.

3.3 Proposed Method: C-SSF1.3

C-SSF1.1 ran a few times faster than C-SSF1.0 without losing the excellent solution quality, but still remained unsatisfactory in processing time for a larger model. C-SSF1.1 needed to be made even faster; thus, this paper proposes a yet faster version C-SSF1.3.

Since the key point is to decrease the number of searches, we decided to put a ceiling S_{max} on the number; however, we should not lose excellent solutions by doing so. All of the limited number S_{max} of searches should start in the promising directions. Note that the decision whether or not this direction will lead to an excellent solution should be made at a starting point. We assumed the larger convex curvature at a starting point, the better solution at the end of the search. See Figure 2.

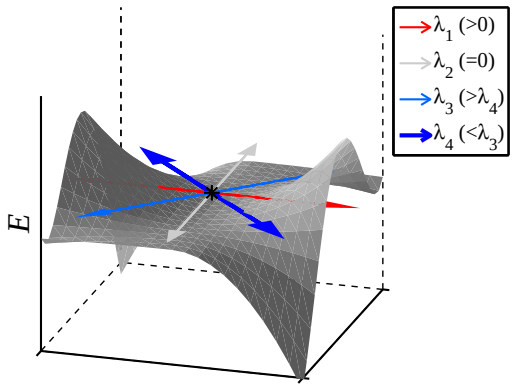


Figure 2: Conceptual diagram of eigenvectors at a point in a singular region.

To implement this, we calculate eigenvalues of all initial points on the singular regions $\widehat{\boldsymbol{\Theta}}_J^{\alpha\beta}$ and $\widehat{\boldsymbol{\Theta}}_J^{\gamma}$. Then, we pick up the limited number of eigenvalues in ascending order, and perform search using their eigenvectors.

One more contrivance was introduced into C-SSF1.3. When C-SSF starts search at one step larger C-MLP(J), only the best solution of C-MLP($J-1$) is used to create the singular regions. Recently we found that the best solution of C-MLP($J-1$) does not always lead to the best solution of C-MLP(J) especially for a very small J. Therefore, we utilize the best R solutions of C-MLP($J-1$) to create the singular regions of C-MLP(J) when $J \le J_R$. Note that when $J \le J_R$, the ceiling on the number of searches is not put. The increase of processing load due to additional searches will be trivial because J is very small.

In the following experiments, C-SSF1.3 system parameters were set as S_{max}= 100, J_R = 3, and R = 3.

4 EXPERIMENTS

The proposed C-SSF1.3 was evaluated using two artificial data sets. That is, the performance of C-SSF1.3 was compared with former version C-SSF1.1, batch-type complex BP with line search (C-BP), and complex BFGS (C-BFGS).

In a C-MLP an activation function plays an important role. We employed the following $\sigma(z)$ (Kim and Guest, 1990; Leung and Haykin, 1991) for a hidden unit. When z is a complex number ($z = a + i\ b$), $\sigma(z)$ is periodic and unbounded.

$$\begin{aligned}\sigma(z) &= \frac{1}{1+e^{-z}} \\ &= \frac{1+e^{-a}\cos b + ie^{-a}\sin b}{1+2e^{-a}\cos b + e^{-2a}} \end{aligned} \quad (13)$$

Real and imaginary parts of initial weights for C-BP and C-BFGS were randomly selected from the range $(-1, 1)$. For each J, C-BP or C-BFGS was performed 100 times changing initial weights.

Each run of any learning method was terminated when the number of sweeps exceeded 10,000 or the step length got smaller than 10^{-16}.

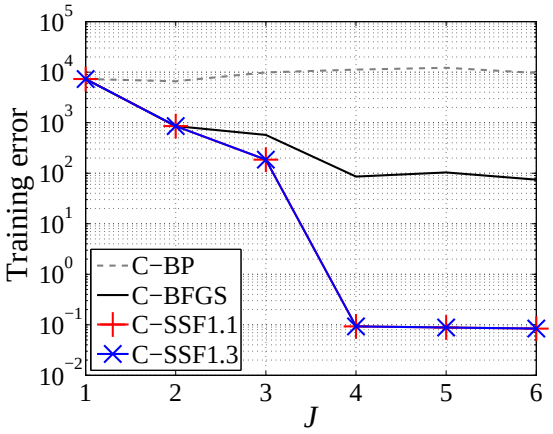
4.1 Experiments using Artificial Data 1

Artificial data 1 was generated using a C-MLP having the following weights with $J = 4$. A PC with Intel(R) Xeon(R) E5-2687W 3.10GHz and 32GB memory was used together with MATLAB2014a.

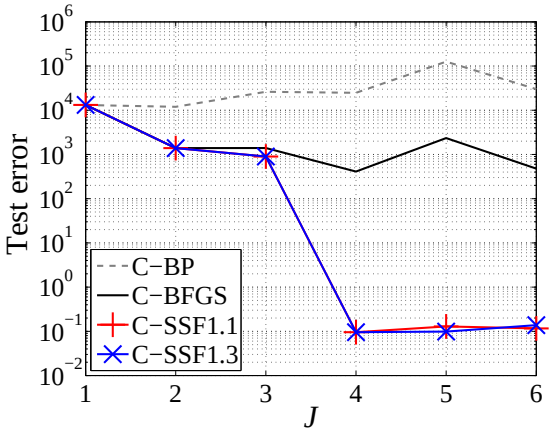
$$\begin{aligned}&(w_0, w_1, w_2, w_3, w_4) \\ &= (-3+1i, -1+1i, 1+1i, 0+5i, 5-4i), \\ &(\boldsymbol{w}_1, \boldsymbol{w}_2, \boldsymbol{w}_3, \boldsymbol{w}_4) \\ &= \begin{pmatrix} -2+3i & 0-5i & -4-5i & -1+1i \\ 4+0i & -2+2i & -1+2i & -2+2i \\ -3+1i & 0-2i & -4-4i & 4+1i \\ 4+4i & 1+4i & 3+0i & -4-1i \\ 0-5i & 5+3i & -1-5i & 3-1i \\ -5-2i & -4+2i & 3-5i & 5+4i \end{pmatrix} \end{aligned} \quad (14)$$

The real and imaginary parts of input x_k were randomly selected from the range $(0, 1)$. Teacher signal y^μ was generated by adding small Gaussian noise $\mathcal{N}(0, 0.01^2)$ to both real and imaginary parts of the output. The size of training data was 500 ($N = 500$), and the maximum number of hidden units was set to 6 ($J_{max} = 6$). Test data of 1,000 data points without noise was generated independently of training data.

Figures 3 (a) and (b) show minimum training error and the corresponding test error respectively. C-BP could not decrease training error and showed very poor generalization. C-BFGS basically decreased training error as J got larger; however, its test error showed the slight up-and-down movement. Both fast versions of C-SSF showed much the same results for training and test, much better than those of C-BFGS for $J \geq 4$. Note also that C-SSF monotonically decreased training error. Both versions of C-SSF and C-BFGS minimized test error at $J = 4$, which is correct.



(a) Training error.



(b) Test error.

Figure 3: Training and test errors for artificial data 1.

Table 1 shows the number of searches for artificial data 1. The numbers of each C-SSF include the ones of pruned searches. Note that the numbers of each C-SSF for $J = 2$ or 3 were larger than 100 because multiple best solutions were utilized to create starting points. The total number of C-SSF1.3 was 29 % (0.71=791/1108) smaller than that of C-SSF1.1.

Table 2 shows CPU time required by each method for artificial data 1. C-BP spent the

Table 1: Numbers of searches for artificial data 1.

J	C-BP	C-BFGS	C-SSF 1.1	C-SSF 1.3
1	100	100	38	38
2	100	100	132	132
3	100	100	321	321
4	100	100	160	100
5	100	100	220	100
6	100	100	237	100
total	600	600	1108	791

longest time 443 minutes in total, which may mean it easily got stuck in poor local minima, and could not escape from them. C-SSF1.3 was 1.23 (=742/602) times faster than C-SSF1.1, and C-BFGS was in the middle of the two. CPU time required by C-BFGS increased as J got larger, while CPU time of C-SSF at $J = 3$ was a bit large due to using multiple best solutions for creating starting points.

Table 2: CPU time for artificial data 1. (hr:min:sec)

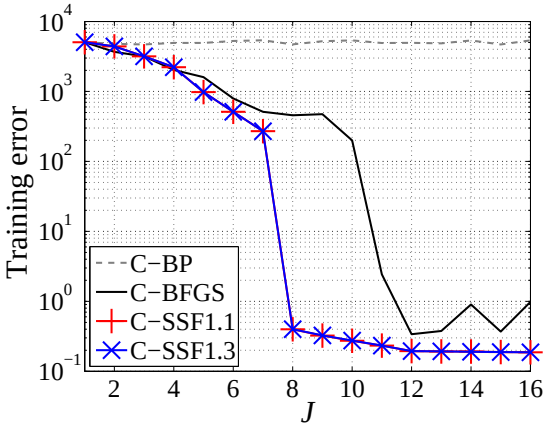
J	C-BP	C-BFGS	C-SSF 1.1	C-SSF 1.3
1	0:35:38	0:00:30	0:00:13	0:00:13
2	0:53:57	0:00:53	0:00:39	0:00:39
3	1:10:27	0:01:08	0:03:09	0:03:09
4	1:39:13	0:02:18	0:01:19	0:01:00
5	1:22:45	0:02:35	0:02:56	0:02:07
6	1:41:25	0:03:54	0:04:06	0:02:53
total	7:23:24	0:11:19	0:12:22	0:10:02

4.2 Experiments using Artificial Data 2

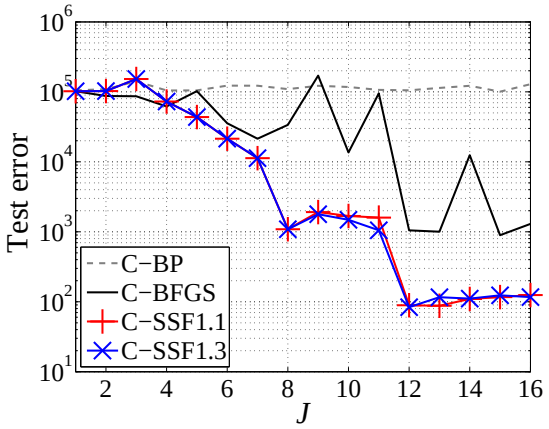
Artificial data 2 was generated using the following logarithmic spirals. How flexibly C-MLP can represent this heavily swirling function was evaluated. XPS 8300 with Intel(R) Core i7-2600 3.40GHz and 12GB memory was used together with MATLAB2014a.

$$\begin{aligned} y &= \{0.001e^{0.1\phi} + 2.5e^{-0.1\phi} + 0.1e^{0.05\phi}\} \\ &\quad \{e^{2i\phi} + e^{5i(\phi+\pi/3)} + e^{12i\phi} + e^{15i\phi}\}, \\ &\quad \text{where} \quad \phi = 2\pi x \end{aligned} \tag{15}$$

The real part of input x^μ was randomly selected from the range $(0, 10)$, and the imaginary part was set to zero. Teacher signal y^μ was generated by adding small Gaussian noise $\mathcal{N}(0, 0.01^2)$ to both real and imaginary parts of the output. The size of training data was 1,000 (N = 1,000), and the maximum number of hidden units was set to 16 (J_{max} = 16). Test data of 1,000 data points without noise was generated from the range $(10, 13)$ of input x, outside of the range of training.



(a) Training error.



(b) Test error.

Figure 4: Training and test errors for artificial data 2.

Figures 4 (a) and (b) show minimum training error and the corresponding test error respectively. Again C-BP could hardly decrease training error and showed very poor generalization. C-BFGS basically decreased training error as J increased, but fluctuated for $J \geq 12$. Both versions of C-SSF showed almost equivalent results for training and test, monotonically decreasing training error. Both C-SSF versions indicate J = 12 or 13 may be the best model.

Table 3 shows the number of searches of each method for artificial data 2. The numbers of each C-SSF include the ones of pruned searches. The

numbers of each C-SSF for $J = 3$ were larger than 100 because multiple best solutions were utilized for $J \leq 3$. The total number of C-SSF1.3 was one-fifth (0.20=1509/7409) of that of C-SSF1.1.

Table 3: Numbers of searches for artificial data 2.

J	C-BP	C-BFGS	C-SSF 1.1	C-SSF 1.3
1	100	100	16	16
2	100	100	81	81
3	100	100	162	162
4	100	100	70	70
5	100	100	80	80
6	100	100	177	100
7	100	100	190	100
8	100	100	269	100
9	100	100	568	100
10	100	100	306	100
11	100	100	593	100
12	100	100	583	100
13	100	100	1042	100
14	100	100	770	100
15	100	100	1664	100
16	100	100	838	100
total	1600	1600	7409	1509

Table 4 shows CPU time required by each method for artificial data 2. C-BP spent the longest CPU time about 41 hours in total. CPU time of C-BFGS gradually increased as J got larger, spending 3.6 hours in total. C-SSF1.3 was the fastest, 3.2 times (=297/94) faster than C-SSF1.1, and 2.3 times (=215/94) faster than C-BFGS. Note that C-SSF1.3 spent about 6 or 7 minutes for each J (≥ 6) except J=13, while C-SSF1.1 showed a tendency to require more CPU time as J got larger.

Figures 5 (a), (b), (c) and (d) show the output of the best models learned by each learning method. The best model means C-MLP(J) minimizing test error; J=15 for C-BP and C-BFGS, J=13 for C-SSF1.1, and J=12 for C-SSF1.3. C-BP could hardly fit the function for the range $(0, 10)$ and showed very poor generalization for the range $(10, 13)$. C-BFGS nicely fitted the function in the range $(0, 10)$, but the amplitude fitting got slightly deviated for the range $(10, 13)$. Both versions of C-SSF very nicely fitted the swirling function all over the range $(0, 13)$ showing excellent generalization.

Table 4: CPU time for artificial data 2. (hr:min:sec)

J	C-BP	C-BFGS	C-SSF 1.1	C-SSF 1.3
1	0:38:58	0:01:03	0:00:10	0:00:10
2	1:11:01	0:01:46	0:02:16	0:02:18
3	1:12:26	0:03:38	0:05:27	0:05:33
4	1:29:03	0:04:47	0:03:00	0:03:07
5	1:39:42	0:06:10	0:03:17	0:03:25
6	1:55:47	0:07:22	0:08:27	0:07:08
7	2:11:15	0:09:22	0:09:16	0:06:26
8	2:22:59	0:11:08	0:14:34	0:07:46
9	2:33:36	0:13:54	0:22:46	0:06:55
10	2:54:17	0:15:51	0:13:31	0:06:27
11	3:04:18	0:18:23	0:30:37	0:07:39
12	3:20:16	0:19:35	0:24:39	0:07:27
13	3:49:26	0:22:03	0:38:24	0:09:32
14	4:00:40	0:26:08	0:24:32	0:06:28
15	4:12:06	0:25:33	1:04:22	0:06:42
16	4:33:26	0:28:11	0:31:31	0:07:02
total	41:09:15	3:34:55	4:56:48	1:34:04

5 CONCLUSION

C-SSF is a completely new learning method for a complex-valued MLP, making good use of singular regions to stably and successively find excellent solutions. We proposed C-SSF1.3 which puts a ceiling on the search load for larger models and utilizes multiple best solutions for smaller models. Although the former versions of C-SSF were rather slow, the proposed C-SSF1.3 ran very fast without losing excellent solution quality. It ran 3.2 times faster than C-SSF1.1, 2.3 times faster than C-BFGS for a larger problem. In the future we plan to apply the method to challenging applications.

ACKNOWLEDGEMENTS

This work was supported by Grants-in-Aid for Scientific Research (C) 25330294 and Chubu University Grant 26IS19A.

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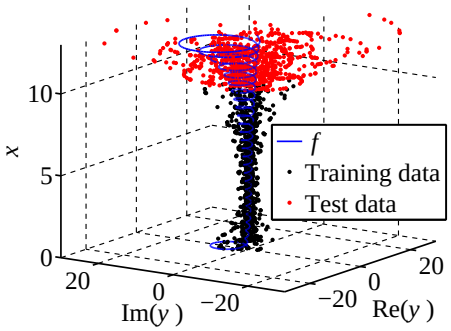
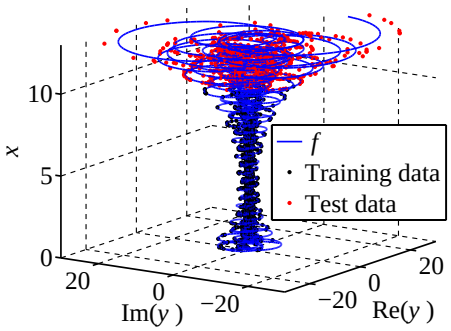
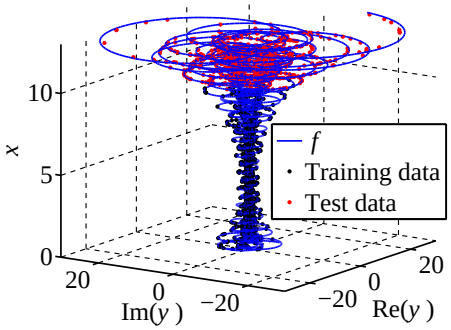


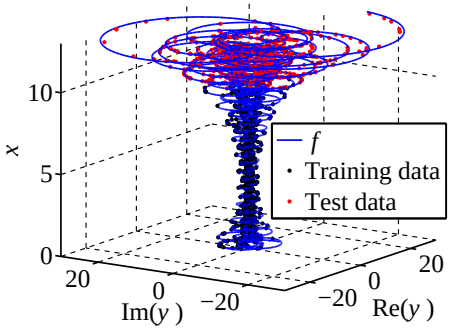
(a) C-BP($J = 15$)



(b) C-BFGS($J = 15$)



(c) C-SSF1.1($J = 13$)



(d) C-SSF1.3($J = 12$)

Figure 5: Outputs of C-MLPs.